An improved simplified numerical model for simulating the rocking response of free-standing objects

Hanquan Liu[1], Zhe Qu[2]*, Yuli Huang[3]

[1]*PhD student, Institute of Engineering Mechanics, CEA, liu_hanquan@163.com*
[2]*Professor, Institute of Engineering Mechanics, CEA, quz@iem.ac.cn*
[3]*Senior Analyst, Arup, yuli.huang@arup.com*

Abstract— **This paper introduces an improved simplified numerical model which adopts local damping to simulate the rocking response of a rigid block. The improved simplified numerical model is implemented in OpenSees and is compared with a conventional model incorporating global viscous damping in simulating existing tests of free rocking objects. The results show that the improved simplified numerical model can better simulate the rocking response.**

I. INTRODUCTION

The seismic response of a solitary rigid block that rocks on a rigid base is typically addressed using the framework proposed by Housner more than half a century ago [1]. Over the years, many researchers have completed numerical simulation of rocking response[3][7][8][9][11][12][13], and many have completed corresponding experiments[2][4][5][6].

The equivalent single degree-of-freedom (SDOF) models with a negative-stiffness rotational spring at the base of a beam element with properties consistent with the geometry of the block are presented by Diamantopoulos and Fragiadakis to simulate the rocking response of a rigid block[11]. A damping force which is assumed continuous and proportional to the velocity is adopted when considering energy dissipation and damping. There are also many models adopted continuous damping (global damping), although they are not consistent with reality (local damping) when it is assumed that energy is only dissipated during impact[6][7].

This paper introduces an improved simplified numerical model which adopts local damping to simulate the rocking response of a rigid block. The improved simplified numerical model is implemented in OpenSees and is compared with a conventional model incorporating global viscous damping in simulating existing tests of free rocking objects.

II. SIMPLIFIED NUMERICAL MODEL

A. The rigid block

The homogeneous rectangular rigid block has dimensions $2b \times 2h$, mass m, and its moment of inertia about the pivot point O, or O', is $I_0 = (4/3)mR^2$(Fig.1.). We assume that the coefficient of friction between the block and its rigid base is always big enough so that the block does not slide. $\alpha = \mathrm{atan}(b/h)$ is the block slenderness angle and $R = \sqrt{b^2 + h^2}$ is its size parameter[1]. The block will start a rocking motion only if the ground acceleration $\ddot{u}_g$ exceeds a threshold value, ie, when $\ddot{u}_g \geq g\tan\alpha = (b/h)g$,where g is the acceleration of gravity.

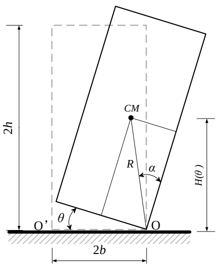

Fig.1. Geometry of the rigid block

B. SDOF model

The equivalent SDOF model is shown in Fig.2.If the seismic force is $F_{eq} = -m\ddot{u}_g$, only the component $F_{eq}\cos\alpha = -m\ddot{u}_g\cos\alpha$ is considered in the model.The SDOF model (Fig.3.) with a nonlinear rotational spring that has a negative stiffness moment-rotation relationship (Fig.4.) is presented[11].we assume that all the mass m is lumped at node 3, and the lumped mass also has a rotational moment of inertia $(mR^2)/3$. The "yield" moment of the nonlinear spring should be equal to $M_0 = mgR\sin\alpha$, which is the moment required for setting a rectangular block from its rest position to a rocking motion. The moment at $\theta = 0$ is M_0, but once the block is set to rocking motion, the restoring moment decreases (negative stiffness) reaching a zero moment at $\theta = \alpha$ (overturning). For simplicity and convenience, this paper adopts the linear M-θ relationship to the bottom spring. The first branch of the available material models is linear elastic and is followed after yielding by a hardening (or a softening) branch. Since our M-θ relationship is not zero at $\theta = 0$, we assume a very small "yield" rotation,$\delta\alpha$.

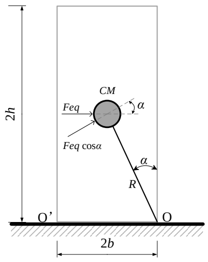

Fig.2. SDOF model for rigid block modelling

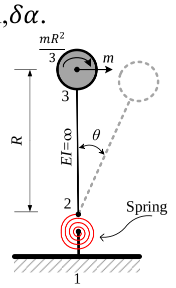

Fig.3. SDOF model

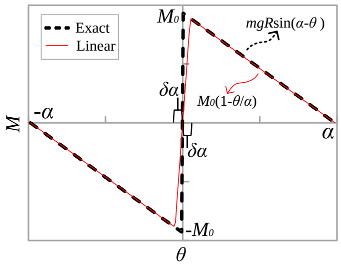

Fig.4. Moment-rotation relationships of the nonlinear rotational spring

C. Local damping and global damping

Housner developed the concept of the coefficient of restitution (η) to idealize the energy loss that occurs during rocking[1], this concept is not easy to implement in finite element software. Some researchers use equivalent viscous damping instead of the coefficient of restitution,however, there are difficulties to determine the appropriate damping ratio. The above equivalent viscous damping are all global damping, which means, assuming that energy dissipation occurs during the entire rocking process. This paper presents a SDOF model with local damping based on ElasticBilinDamped material which is proposed by Qu in OpenSees, which means, energy dissipation occurs only during "yield" rotation($\delta\alpha$), not the entire rocking process.

The equation of motion describing the free rocking behavior of the SDOF model with local damping is

$$\begin{cases} I_0\ddot{\theta} + c\dot{\theta} + k\theta = 0, & -\delta\alpha < \theta < \delta\alpha \\ I_0\ddot{\theta} + k\theta = 0, & \text{otherwise} \end{cases} \tag{1}$$

The damping coefficient c is obtained with the aid of the principle of conservation of angular momentum as

$$I_0\dot{\theta}_1 - c\bar{\dot{\theta}}\frac{2\delta\alpha}{\bar{\dot{\theta}}} = I_0\dot{\theta}_2 \tag{2}$$

Where $\bar{\dot{\theta}}$ is average angular velocity during "yield" rotation($\delta\alpha$).

$$c = \frac{2mR^2}{3\delta\alpha}(1-\eta)|\dot{\theta}_1| = c'|\dot{\theta}_1| \tag{3}$$

Where η is the coefficient of restitution proposed by Housner.

A stiffness-proportional Rayleigh model is used as global damping model in this paper. The damping ratio of SDOF model with global damping is calibrated by rocking experimental which had been completed by Aslam et al[2].

Two models with different damping types are used to simulate free rocking response of a rigid block. The moment-rotation relationships of two models are depicted in Fig.5.For the model with local damping, obviously , energy dissipation occurs only during "yield" rotation($\delta\alpha$).When the rotation θ beyond the "yield" rotation($\delta\alpha$),there is no energy dissipation. However, for the model with global damping, energy dissipation occurs during the entire rocking process.

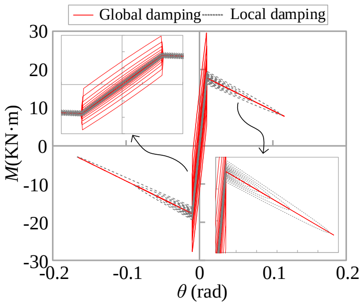

Fig.5. Moment-rotation relationships

III. Comparison between Model and Test

The above two models are used to simulate free rocking and compared with rocking experimental which had been completed by Aslam et al[2]. The results are depicted in Fig.6. The local damping model can better simulate the free rocking of the test, compared with the global damping model.

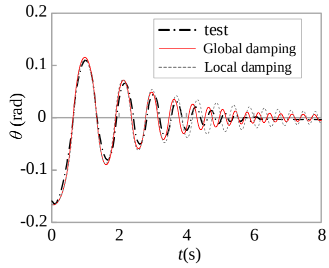

Fig.6. Free rocking time history

IV. Conclusions

(1) The SDOF model with a nonlinear rotational spring that has a negative stiffness moment-rotation relationship can simulate the rocking response of a rigid block well.

(2) Local damping model is more in line with reality than global damping model when it is assumed that energy is only dissipated during impact.

References

[1] Housner G W. The behavior of inverted pendulum structures during earthquakes[J]. Bulletin of the seismological society of America, 1963, 53(2): 403-417.

[2] Aslam M. Earthquake rocking response of rigid bodies[J]. 1978.

[3] Zhang J, Makris N. Rocking response of free-standing blocks under cycloidal pulses[J]. Journal of Engineering Mechanics, 2001, 127(5): 473-483.

[4] Filiatrault A, Kuan S, Tremblay R. Shake table testing of bookcase–partition wall systems[J]. Canadian Journal of Civil Engineering, 2004, 31(4): 664-676.

[5] Konstantinidis D, Makris N. Experimental and analytical studies on the response of 1/4-scale models of freestanding laboratory equipment subjected to strong earthquake shaking[J]. Bulletin of earthquake engineering, 2010, 8(6): 1457-1477.

[6] Nasi K T J. Stability of rocking structures[M]. Purdue University, 2011.

[7] Vassiliou M F, Mackie K R, Stojadinović B. Dynamic response analysis of solitary flexible rocking bodies: modeling and behavior under pulse‐like ground excitation[J]. Earthquake engineering & structural dynamics, 2014, 43(10): 1463-1481.

[8] Dimitrakopoulos E G, Paraskeva T S. Dimensionless fragility curves for rocking response to near‐fault excitations[J]. Earthquake engineering & structural dynamics, 2015, 44(12): 2015-2033.

[9] Fragiadakis M, Psycharis I, Cao Y, et al. Parametric investigation of the dynamic response of rigid blocks subjected to synthetic near-source ground motion records[C]//ECCOMAS Congress, Crete Island, Greece. 2016: 5-10.

[10] Petrone C, Di Sarno L, Magliulo G, et al. Numerical modelling and fragility assessment of typical freestanding building contents[J]. Bulletin of Earthquake Engineering, 2017, 15(4): 1609-1633.

[11] Diamantopoulos S, Fragiadakis M. Seismic response assessment of rocking systems using single degree‐of‐freedom oscillators[J]. Earthquake Engineering & Structural Dynamics, 2019, 48(7): 689-708.

[12] Fragiadakis M, Diamantopoulos S. Risk assessment of rocking contents in Multistorey buildings[C]//16ECEE, 16th European Conference on Earthquake Engineeering, At Thessaloniki, Greece. 2018.

[13] Tomassetti U, Graziotti F, Sorrentino L, et al. Modelling rocking response via equivalent viscous damping[J]. Earthquake Engineering & Structural Dynamics, 2019, 48(11): 1